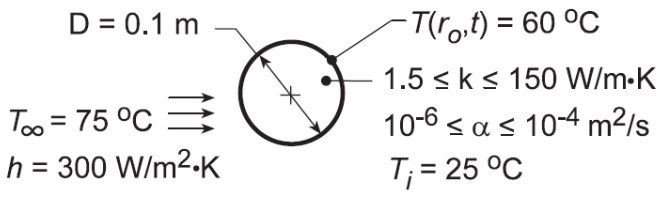
PROBLEM 08

KNOWN: Initial temperature and properties of a solid sphere. Surface temperature after immersion in a fluid of prescribed temperature and convection coefficient.

FIND: (a) Time to reach surface temperature, (b) Effect of thermal diffusivity and conductivity on thermal response.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties.

ANALYSIS: (a) For k = 15 W/m·K, the Biot number is

$$\mathrm{Bi} = \frac{\mathrm{h}\left(\mathrm{r_o}/3\right)}{\mathrm{k}} = \frac{300\,\mathrm{W/m^2\cdot K}\left(0.05\,\mathrm{m}/3\right)}{15\,\mathrm{W/m\cdot K}} = 0.333.$$

Hence, the lumped capacitance method cannot be used. From Equation 5.53a,

$$\frac{\mathrm{T}-\mathrm{T}_\infty}{\mathrm{T_i}-\mathrm{T}_\infty} = \mathrm{C_1}\exp\left(-\zeta_1^2\mathrm{Fo}\right)\frac{\sin\left(\zeta_1 \mathrm{r}^*\right)}{\zeta_1 \mathrm{r}^*}.$$

At the surface, $r^* = 1$. From Table 5.1, for Bi = 1.0, $\zeta_1 = 1.5708$ rad and $C_1 = 1.2732$. Hence,

$$\frac{60-75}{25-75} = 0.30 = 1.2732\exp\left(-1.5708^2\,\mathrm{Fo}\right)\frac{\sin 90^\circ}{1.5708}$$

$$\exp(-2.467\mathrm{Fo}) = 0.370$$

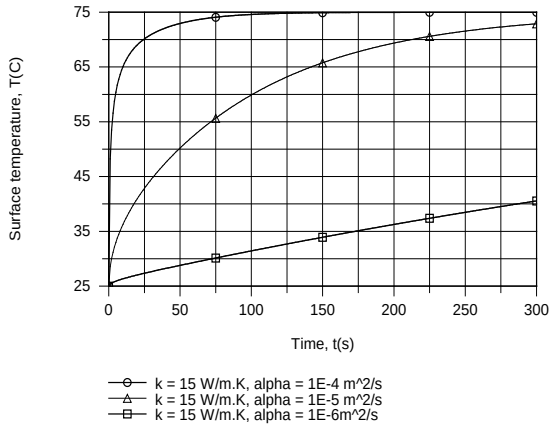
$$\mathrm{Fo} = \frac{\alpha \mathrm{t}}{\mathrm{r_o^2}} = 0.403$$

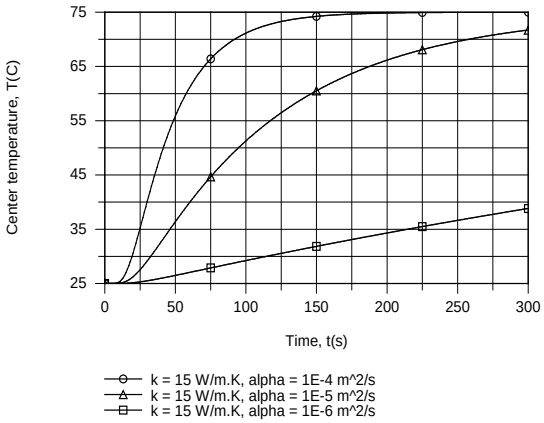
$$\mathrm{t} = 0.403\frac{\mathrm{r_o^2}}{\alpha} = 0.403\frac{\left(0.05\,\mathrm{m}\right)^2}{10^{-5}\,\mathrm{m^2/s}} = 100\mathrm{s}$$ <

(b) Using the IHT *Transient Conduction Model* for a *Sphere* to perform the parametric calculations, the effect of α is plotted for k = 15 W/m·K.

Continued...

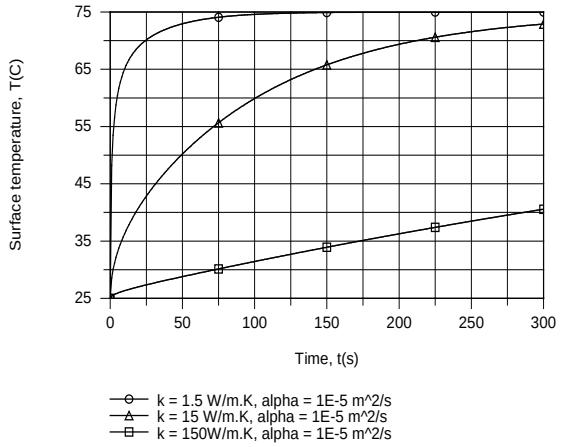
PROBLEM 08 (Cont.)

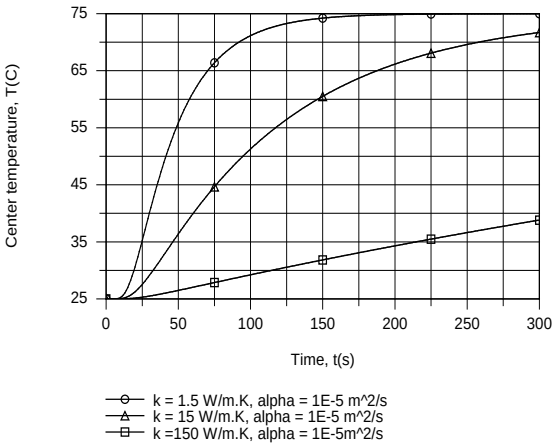




For fixed k and increasing α, there is a reduction in the thermal capacity (ρc_p) of the material, and hence the amount of thermal energy which must be added to increase the temperature. With increasing α, the material therefore responds more quickly to a change in the thermal environment, with the response at the center lagging that of the surface.

The effect of k is plotted for $\alpha = 10^{-5}$ m^2/s.





With increasing k for fixed alpha, there is a corresponding increase in ρc_p, and the material therefore responds more slowly to a thermal change in its surroundings. The thermal response of the center lags that of the surface, with temperature differences, $T(r_o,t) - T(0,t)$, during early stages of solidification increasing with decreasing k.

COMMENTS: Use of this technique to determine h from measurement of $T(r_o)$ at a prescribed t requires an iterative solution of the governing equations.