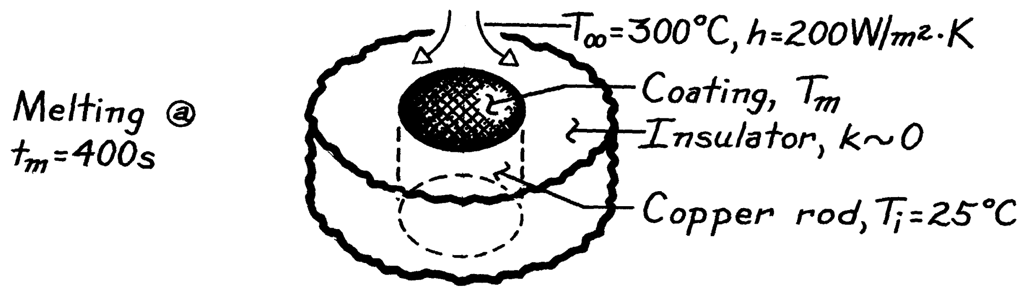
PROBLEM 09

KNOWN: Procedure for measuring convection heat transfer coefficient, which involves melting of a surface coating.

FIND: Melting point of coating for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in solid rod (negligible losses to insulation), (2) Rod approximated as semi-infinite medium, (3) Negligible surface radiation, (4) Constant properties, (5) Negligible thermal resistance of coating.

PROPERTIES: Copper rod (Given): k = 400 W/m·K, $\alpha = 10^{-4}$ m^2/s.

ANALYSIS: Problem corresponds to transient conduction in a semi-infinite solid. Thermal response is given by

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2(\alpha t)^{1/2}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\right]\left[\operatorname{erfc}\left(\frac{x}{2(\alpha t)^{1/2}} + \frac{h(\alpha t)^{1/2}}{k}\right)\right].$$

For x = 0, erfc(0) = 1 and T(x,t) = T(0,t) = T_s. Hence

$$\frac{T_s - T_i}{T_\infty - T_i} = 1 - \exp\left(\frac{h^2 \alpha t}{k^2}\right)\operatorname{erfc}\left(\frac{h(\alpha t)^{1/2}}{k}\right)$$

with

$$\frac{h(\alpha t_m)^{1/2}}{k} = \frac{200\ \text{W/m}^2\cdot\text{K}\left(10^{-4}\text{m}^2/\text{s}\times 400\ \text{s}\right)^{1/2}}{400\ \text{W/m}\cdot\text{K}} = 0.1$$

$$T_s = T_m = T_i + \left(T_\infty - T_i\right)\left[1 - \exp(0.01)\operatorname{erfc}(0.1)\right]$$

$$T_s = 25^\circ\text{C} + 275^\circ\text{C}\left[1\text{-}1.01\times 0.888\right] = 53.5^\circ\text{C}.$$ <

COMMENTS: Use of the procedure to evaluate h from measurement of t_m necessitates iterative calculations.