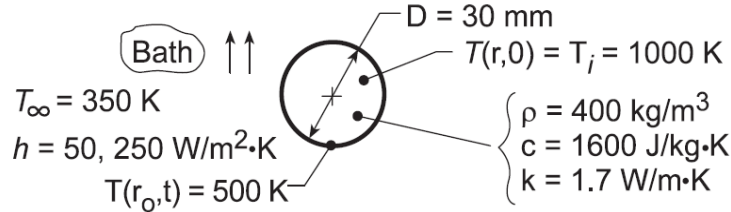


PROBLEM 05

KNOWN: A long cylinder, initially at a uniform temperature, is suddenly quenched in a large oil bath.

FIND: (a) Time required for the surface to reach 500 K, (b) Effect of convection coefficient on surface temperature history.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) $Fo > 0.2$.

ANALYSIS: (a) Check first whether lumped capacitance method is applicable. For $h = 50 \text{ W/m}^2\cdot\text{K}$,

$$Bi_c = \frac{hL_c}{k} = \frac{h(r_o/2)}{k} = \frac{50 \text{ W/m}^2 \cdot \text{K} (0.015 \text{ m} / 2)}{1.7 \text{ W/m} \cdot \text{K}} = 0.221.$$

Since $Bi_c > 0.1$, method is not suited. Using the approximate series solution for the infinite cylinder,

$$\theta^*(r^*, Fo) = C_1 \exp(-\zeta_1^2 Fo) \times J_0(\zeta_1 r^*) \quad (1)$$

Solving for Fo and setting $r^* = 1$, find

$$Fo = -\frac{1}{\zeta_1^2} \ln \left[\frac{\theta^*}{C_1 J_0(\zeta_1)} \right]$$

$$\text{where } \theta^* = (1, Fo) = \frac{T(r_o, t_o) - T_\infty}{T_i - T_\infty} = \frac{(500 - 350) \text{ K}}{(1000 - 350) \text{ K}} = 0.231.$$

From Table 5.1, with $Bi = 0.441$, find $\zeta_1 = 0.8882 \text{ rad}$ and $C_1 = 1.1019$. From Table B.4, find $J_0(\zeta_1) = 0.8121$. Substituting numerical values into Eq. (2),

$$Fo = -\frac{1}{(0.8882)^2} \ln [0.231 / 1.1019 \times 0.8121] = 1.72.$$

From the definition of the Fourier number, $Fo = \alpha t / r_o^2$, and $\alpha = k / \rho c$,

$$t = Fo \frac{r_o^2}{\alpha} = Fo \cdot r_o^2 \frac{\rho c}{k}$$

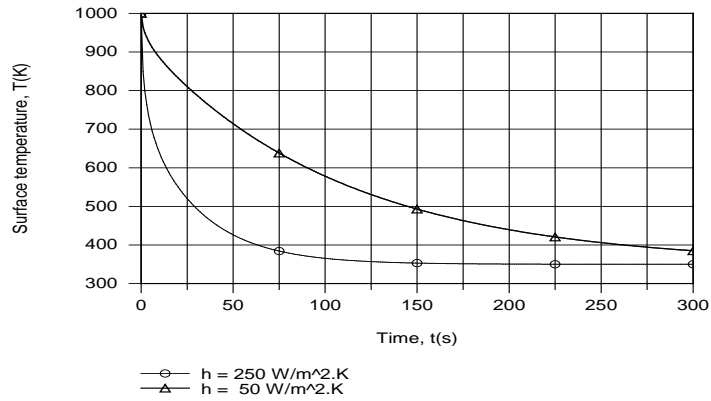
$$t = 1.72 (0.015 \text{ m})^2 \times 400 \text{ kg/m}^3 \times 1600 \text{ J/kg} \cdot \text{K} / 1.7 \text{ W/m} \cdot \text{K} = 145 \text{ s}.$$

<

(b) Using the IHT *Transient Conduction Model* for a *Cylinder*, the following surface temperature histories were obtained.

Continued...

PROBLEM 05 (Cont.)



Increasing the convection coefficient by a factor of 5 has a significant effect on the surface temperature, greatly accelerating its approach to the oil temperature. However, even with $h = 250 \text{ W/m}^2\cdot\text{K}$, $Bi = 1.1$ and the convection resistance remains significant. Hence, in the interest of accelerated cooling, additional benefit could be achieved by further increasing the value of h .

COMMENTS: For Part (a), note that, since $Fo = 1.72 > 0.2$, the approximate series solution is appropriate.