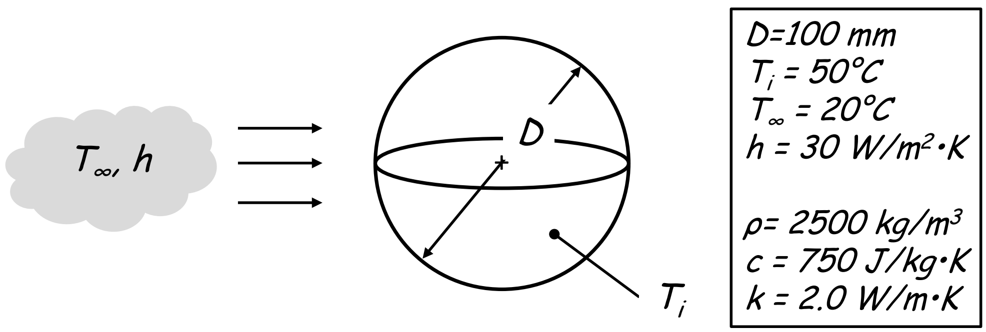
PROBLEM 07

KNOWN: Properties, diameter, and initial temperature of sphere. Environment temperature and heat transfer coefficient.

FIND: Time for sphere's surface temperature to reach 26°C. Energy transferred from sphere up to that time.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional (radial) conduction in sphere. (2) Uniform properties. (3) Negligible radiation.

PROPERTIES: *Given*, ρ = 2500 kg/m^3, c = 750 J/kg·K, k = 2.0 W/m·K.

ANALYSIS: The Biot number is first calculated to determine if the lumped capacitance method is valid.

$$Bi = \frac{h(r_o/3)}{k} = \frac{30\ \text{W/m}^2\cdot\text{K}(0.05\ \text{m/3})}{2.0\ \text{W/m}\cdot\text{K}} = 0.25$$

The lumped capacitance method is not valid and the radial dependence of temperature must be accounted for. We try the one-term approximation and will check the Fourier number later.

$$Bi = \frac{hr_o}{k} = \frac{30\ \text{W/m}^2\cdot\text{K}(0.05\ \text{m})}{2.0\ \text{W/m}\cdot\text{K}} = 0.75$$

From Table 5.1, $\zeta_1 = 1.392$, $C_1 = 1.211$. From the specified temperatures, $\theta^* = (T - T_\infty)/(T_i - T_\infty)$ = 6/30 = 0.2. Then from Equation 5.53a, with r^* = 1,

$$\theta^* = C_1 \exp(-\zeta_1^2 Fo)\frac{1}{\zeta_1}\sin(\zeta_1), \qquad Fo = -\frac{1}{\zeta_1^2}\ln\left(\frac{\theta^*\zeta_1}{C_1\sin(\zeta_1)}\right) = -\frac{1}{1.392^2}\ln\left(\frac{0.2\times 1.392}{1.211\sin(1.392)}\right) = 0.750$$

Since Fo > 0.2, the one-term approximation is valid. Finally,

$$t = \frac{r_o^2 Fo}{\alpha} = \frac{(0.05\ \text{m})^2 \times 0.750}{\left(2.0\ \text{W/m}\cdot\text{K}/(2500\ \text{kg/m}^3 \times 750\ \text{J/kg}\cdot\text{K})\right)} = 1760\ \text{s} \qquad <$$

Continued…

PROBLEM 07 (Cont.)

The energy transferred can be found from Equation 5.55, with $Q_o = \rho c V(T_i - T_\infty)$ and $\theta_o{}^*$ from Equation 5.53c:

$$Q_o = \rho c V(T_i - T_\infty) = 2500 \text{ kg/m}^3 \times 750 \text{ J/kg}\cdot\text{K} \times \left(\frac{4}{3}\pi(0.05 \text{ m})^3\right) \times 30°\text{C} = 29,450 \text{ J}$$

$$\theta_o{}^* = C_1 \exp(-\zeta_1^2 Fo) = 1.211\exp(-(1.392)^2 \times 0.750) = 0.283$$

$$Q = \rho c V(T_i - T_\infty)\left[1 - \frac{3\theta_o{}^*}{\zeta_1^3}\left(\sin\zeta_1 - \zeta_1\cos\zeta_1\right)\right]$$

$$= 29,450 \times \left[1 - \frac{3\times 0.283}{1.392^3}\left(\sin(1.392) - 1.392\cos(1.392)\right)\right] = 22,600 \text{ J}$$

<

COMMENTS: If the Fourier number had been smaller than 0.2, more terms would have been required in the infinite series of Equation 5.51a, and an iterative solution would have been required to find *Fo*.